

# Honors Pre-Calculus/Trig

Course Number: 2111

Course Level: Senior High

Grade: 10-12

Time/Credit: Senior High – 1 semester/90 minutes daily

Central Bucks School District

16 Welden Drive

Doylestown, PA 18901

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**Central Bucks School District**

**K-12 Mathematics Teaching and Learning Principles**

1. Investigating, problem-solving, and communicating about mathematics (speaking, reading, writing, representing, and listening) stimulates students’ curiosity, builds confidence, and leads students toward deeper conceptual understanding.
2. Lessons should address students’ strengths, interests, and learning styles through differentiated instruction.
3. All students can achieve excellence in mathematics when:
* instruction is appropriately paced
* teachers set high expectations
* students feel supported by their teachers, parents and administrators
1. Students must be actively engaged in learning experiences designed to connect to their prior knowledge.
2. For students to transfer and use knowledge meaningfully teachers need to:
	* present new ideas within rich and meaningful contexts
* help students to make connections to their lives both within and outside of school
1. Mathematics instruction is more effective when teachers incorporate:
* manipulatives
* calculators
* technology
* real-time data collection devices (motion detectors, temperature probes)
1. Assessment should be ongoing, diagnostic, and aligned with instruction.

Documents Considered:

(2000). Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press.

Marzano, R., & Pickering, D. (1997). *Dimensions of Learning.* Alexandria,VA: Association for Supervision and Curriculum Development; Aurora, CO: Mid-continent Regional Educational Laboratory.

Tomlinson, Carol Ann. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development.

Zemelman, S., Daniels, H., & Hyde, A. (1998). *Best practice: New Standards for Teaching and Learning in America's Schools*. Portsmouth, NH: Heinemann.

**K – 12 Mathematics Program Enduring Understandings**

**Students will understand that:**

**The Nature of Mathematics**

* Mathematics is a language of carefully defined terms and symbols.
* Mathematics is used to make informed decisions about problems in everyday life.

**Numbers and Operations**

* There are multiple representations for any number.
* Numbers are classified and compared within our number system.
* Depending on the situation, calculations may be done using mental math or paper-and-pencil calculations using a variety of mathematically sound algorithms.
* Estimates help determine the reasonableness of an answer.
* Expressions are simplified using a predetermined order of operations.

**Measurement**

* Measurements collected using any tools (except for counting) are estimates with an accepted level of error.
* All measurements consist of exactly three parts: number, unit, and precision.
* Changing one linear dimension in a figure with two or three dimensions will have a different effect on the figure’s perimeter, circumference, area, or volume.

**Geometry**

* Points, lines, and planes are the building blocks of geometry.
* Each geometric figure has a specific naming convention.
* Postulates, theorems, definitions, and properties are used to:
	+ justify reasoning in a direct proof.
	+ establish relationships involving two and three-dimensional figures.
* Logical thinking and critical reasoning skills are important aspects of problem solving.
* A shape’s characteristics (dimensionality, side measures, angle measures, faces, edges, area, perimeter and volume) are used for identification.
* The Pythagorean Theorem and trigonometric ratios are used to find missing quantities in right triangles.
* Concepts of congruency and similarity are used to relate and compare 2 and 3- dimensional figures.

**Algebraic Concepts**

* Algebraic rules and properties determine how expressions are simplified and how equations are solved.
* Problems involving a constant rate of change can be modeled using proportional reasoning.
* Algebraic expressions, equations, inequalities, and functions (linear, absolute value, quadratic, polynomial, exponential, and logarithmic) are used to model relationships between quantities in real-world situations.
* Patterns and functions can be generalized and represented using verbal models, tables, equations, and graphs.
* Systems of equations are solved both graphically and algebraically and are used to model real-life problems.

**Data Analysis and Probability**

* Data is collected, organized, and displayed for analysis.
* Interpretations and predictions about data are influenced by the method that data is collected and displayed.
* Probability is used to make informed predictions and inferences.

**K – 12 Mathematics Program Essential Questions**

**The Nature of Mathematics**

* What characteristics make mathematics a language?
* When is the correct solution not the answer to a problem?

**Numbers and Operations**

* How do you know that an answer is reasonable?
* Why are there different number systems?
* How do you determine when to calculate with pencil and paper, find an estimate, or use a calculator?
* What does it mean to be simplified?
* Is there more than one way to simplify an expression?

**Measurement**

* What does it mean to be accurate?
* How do you measure?
* How does changing a dimension affect a figure’s length, area and volume?

**Geometry**

* Why do all statements need justification?
* What is the logical progression of statements in a proof?
* How are figures classified?
* How do you name a geometric figure and why are standard naming conventions important?
* Why are points, lines, and planes so important in Euclidean geometry?
* How and when is the Pythagorean Theorem used?
* What do similarity and congruency statements tell you about polygons?

**Algebraic Concepts**

* What does “solve” mean?
* Is your solution always the appropriate answer to a problem?
* How do expressions and equations differ?
* What makes a relationship linear?
* How can a line be used to make predictions?
* When is it necessary to use a system of equations?
* Why would you represent a pattern/function in different ways?

**Data Analysis and Probability**

* What is the purpose of collecting data?
* How is data used or abused?
* How do we make predictions based on probability?

Central Bucks School District – Course of Study Overview

Honors Pre-Calculus/Trig

Desired Results.

Course Description: 2111

This course is an honors course designed for students with outstanding mathematical ability and interest who have mastered the concepts and skills of Algebra 2/Trig. Honors mathematics courses require students to move at a faster pace than the equivalent standard level course and they are different from the equivalent standard level course in both the quality of the work expected and the quantity of the work required inside and outside of the classroom. Students taking Honors Precalculus/Trig will have a more rigorous study of the concepts in preparation for future AP courses in mathematics. Honors Precalculus/Trig is an extension of the concepts covered in the two prerequisite courses with an emphasis on the functional aspects necessary for preparation for the study of calculus. Polynomial, exponential, logarithmic, and trigonometric functions are addressed in this course. Trigonometric topics include the solution of trigonometric equations, identity manipulations, and transforming graphs, including work with amplitude, period, and phase shift.

Conic sections, polar and parametric equations, and limits will be introduced.

Incoming 10th and 11th graders who intend to take AP Calculus AB should take Honors Precalculus/Trig to more fully prepare for the pace and rigor of an AP course. The decision to take Honors Pre-calculus/Trig should not be taken lightly, and this decision should be discussed with your Algebra 2/Trig teacher so there is no question regarding the expectation for the course.

This course is weighted at .25 as a recognition of the fact that it is more demanding and has more requirements that go beyond those of the standard Precalculus/Trig course.

Prerequisite: (2900) Honors Algebra 2/Trig, B- or better and teacher recommendation or (2530) Honors Algebra 2/Trig, B- or better and teacher recommendation or (2520) Algebra 2/Trig, A- or better and teacher recommendation.

Core Assessment 1: Solving Trigonometric Equations

Core Assessment 1 Description: Students will solve trigonometric equations and determine if solutions are correct or extraneous. This core assessment addresses solving trigonometric equations using factoring, trigonometric identities, double angle formulas, and using the unit circle for exact solutions.

Core Assessment 1 Grading Criteria: Assessment is graded on a 4‐point item specific scoring guide. This is done to model the kind of scoring that is used for open‐ended problems on the AP Calculus Exam. This assessment addresses anchors A‐Numbers and Operations and D‐Algebraic Concepts.

Core Assessment 2: Evaluation and Analysis of Limits

Core Assessment 2 Description: Students will evaluate limits and determine continuity using graphical analysis and algebraic techniques. The core assessment addresses algebraically and graphically solving limits and using limits test to determine continuity.

Core Assessment 2 Grading Criteria: Assessment is graded on a 4‐point item specific scoring guide. This is done to model the kind of scoring that is used for open‐ended problems on the AP Calculus Exam. This assessment addresses anchors A‐Numbers and Operations and D‐Algebraic Concepts.

Final Exam:

Final exam grade is 20% of the final course grade. The final exam grade is calculated using the following weights: final exam‐70%, core 1‐15%, core 2‐15%.

Instructional Time

|  |  |  |  |
| --- | --- | --- | --- |
| **Content** | **Section(s)** | **Blocks** | **Presentation** |
| **Unit 1: Algebra 2/Trig Review** and Polys |  | **12-14** |  |
| **Algebra Review**: Factoring, solving quadratic equations, complex numbers, exponents, writing equations of lines, slope and y-intercept, parallel, perpendicular, simplifying radicals, roots of functions, operations & composite function, inverse functions (algebraically, graphically), synthetic & long division | 1.3, 1.8, 1.9, 2.1 and supplement | 2-3 | R,I,E |
| Domain and range, interval notation (graphically, algebraically) | 1.4, 1.5 | 1.5 | R,I,E |
| Parent graphs & transformations (quadratics, cubic, square root, cube root, power functions, absolute value, linear and nonlinear piecewise functions) | 1.6-1.7 | 1.5 | R,E |
| Part 1 Quest |  | 1 |  |
| Degrees of polynomials & end behavior, relative maximum and minimum values, increasing, decreasing, constant behavior, analyzing behavior of graphs (with & w/o a calculator), | 2.1-2.2 | 2 | I,E |
| Factor Theorem, Remainder Theorem and Intermediate Value Theorem | 1.5, 2.3 | 1 | I,E |
| Finding roots & graphing polynomials, Rational Root Test, Descartes Rule of Signs, upper & lower bounds | 2.5 | 2 | I,E |
| Part 2 Quest |  | 1 |  |
| **Unit 2: Exponential and Logarithmic Functions** |  | **11** |  |
| The number e and compound interest applications | 3.1 | 1 | R,E |
| Annuities & Annuity Project | Supplement | 3 | I,E |
| Graphing logarithmic functions & transformations (domain, range, and graphical analysis) | 3.2 | 1 | R,E |
| Evaluating and using properties of logarithms | 3.3 | .5 | R,E |
| Expanding & condensing using properties | 3.3 | .5 | R,E |
| Solving exponential & logarithmic equations | 3.4 | 1.5 | R,E |
| Exponential applications (more complicated equations, solving for variables in the exponent) | 3.5 and throughout Chapter 3 | 1.5 | I,E |
| Review & Test |  | 2 |  |
| **Unit 3: Triangle Trigonometry and the Unit Circle** |  | **9** |  |
| Angles in the coordinate plane (graphing, co-terminal angles, radians & degrees, references angles) | 4.1, 4.4 | 1 | I,E |
| Right triangle trigonometry, Unit Circle (derivative and connection to special right triangles) & exact values | 4.2, 4.3 | 2 | R, I, E |
| Non-Unit Circle angles | 4.4 | 1 | I,E |
| Law of Sines and Law of Cosines  | 6.1, 6.2 | 2 |  |
| Applications of triangle trigonometry | 4.3, 6.1, 6.2 | 1 | R,I,E |
| Review & Test |  | 2 |  |
| **Unit 4: Graphing Trigonometric Functions (Radians Only)** |  | **8-9** |  |
| Graphing Sine and Cosine and writing equations | 4.5 | 3 | I, E |
| Graphing the Secant and Cosecant and writing equations | 4.6 | 1 | I, E |
| Graphing Tangent and Cotangent and writing equations | 4.6 | 1 | I,E |
| Graphing the Inverse Trig Functions (Parent Only)  | 4.7 | 1 | I |
| Applications of trigonometric graphs/equations | 4.8 and supplement | 1 | I,E |
| Review & Test |  | 2 |  |
| **Unit 5: Trigonometric Identities & Solving Trigonometric Equations** |  | **10-12** |  |
| Reciprocal, Quotient and Pythagorean Identities | 5.1 | 1 | I,E |
| Simplifying and verifying trigonometric identities  | 5.1-5.2 | 4-5 | I,E |
| Solving trigonometric equations | 5.3 | 2 | I,E |
| Sum and difference formulas, multiple-angle formulas, and half-angle formulas with verifying and solving | 5.4-5.5 | 2 | I,E |
| Review & Test |  | 2 |  |
| **Unit 6: Conic Sections** |  | **10** |  |
| Completing the Square and the Circle | 10.3 | 1 | I,E |
| The Ellipse | 10.3 | 2 | I,E |
| The Hyperbola | 10.4 | 2 | I,E |
| The Parabola | 10.2 | 2 | I,E |
| Identifying Conic Sections | 10.4 and supplement | Throughout | I,E |
| Review & Test |  | 2 |  |
| **Unit 7: Parametric and Polar** |  | **7-8** |  |
| Parametric Equations | 10.6 | 2 |  |
| Polar Coordinates | 10.7 | 1 |  |
| Graphing Polar Equations | 10.8 | 2 |  |
| Review and Test |  | 2 |  |
| **Unit 8: Limits** |  | **9-10** |  |
| Rational Functions (Asymptotes, Holes, Graphing, Writing Equations) | 2.6 | 1 | R,E |
| Definition of a Limit, Graphical Analysis of Limits, One Sided Limits, Graphing Piecewise Functions | 12.1 | 2-3 | I,E |
| Algebraic Techniques: simplification, substitution, factoring, trig limits (direct substitution only), calculator table, conjugate method, complex fractions  | 12.1-12.2 | 2-3 | I,E |
| Limits at Infinity, Graphical/Algebraic Analysis  | 12.4 | 1 | I,E |
| Continuity | Supplement | 1.5 | I,E |
| Criteria Graphing | Supplement | Optional | I,E |
| Review and Test |  | 2 |  |
| Total Number of Days  |  | **76-83days** |  |

**Required and Supplementary Texts/Software**

Textbook Recommendation:

Larson, et. al. (2007). Precalculus with Limits. Boston, Mass.: Houghton-Mifflin. ISBN 978-0-618-75313-0

Geometer’s Sketchpad: Computer software especially helpful with the graphing elements of this course.

Quia.com: This online quiz, survey and activity website can be helpful for review and assessment.

Microsoft OneNote: This online interactive digital notebook can be helpful for taking and organizing notes, review, discussions, class collaboration, and assessment.

Desmos: Online digital graphing calculator that can be helpful with graphing function and exploring various concepts in the course.

**Graphing Calculator – The graphing calculator is an essential part of this course. It should be used on a daily basis. All students need to have access to a graphing calculator for this course.**

Calculator Recommendation:

The secondary mathematics department of Central Bucks School District recognizes the use of calculators as a valuable tool for learning in the mathematics classroom. Graphing calculators will be used daily during this course. In certain advanced courses, graphing calculators with specific capabilities are important for daily classroom performance and are required for advanced placement tests. While no specific brands are endorsed, there are restrictions on the type of calculators allowed on certain classroom tests and final exams. The teacher will use the TI‐83/84 graphing calculator daily during their instruction.

Central Bucks School District – Course of Study

**Unit 1: Algebra 2 Review**

**Desired Results**

**Essential Questions:**

* How do you determine if a relation is a function?
* Why is it that when you take the composition of inverses you always get x?
* Why is the order of the functions important when dealing with operations of functions?
* Why is it important to know the domain of a function?
* What is the connection with solving inequalities and interval notation?
* How do the transformations of a polynomial function change the parent graph?
* Why is the degree of a polynomial important?
* What is the most effective method of solving polynomials?
* What is the relationship between the roots, zeros, solutions, factors, and x-intercepts of a polynomial function?
* How can polynomial functions be used to model real-world situations?

**Enduring Understandings:**

*Students will understand…*

* the difference between a relation and a function.
* how to identify domain given the graph or the equation using proper interval notation.
* the characteristics of parent graphs.
* how to find the inverse of a function and understand its reflective properties.
* why we use the vertical and horizontal line tests.
* how to combine functions using basic operations and also compositions.
* how the changes in values in an equation manipulate the parent graphs.
* by determining all of the characteristics of the function they can graph the function.
* how the degree of a polynomial affects the end behavior of a graph.
* there is a connection/relationship between zeros, solutions, roots, x-intercepts and factors.
* there are various ways of solving for the zeros of a polynomial function (including synthetic and long division).
* solutions to polynomial functions can be real or imaginary.
* the degree of the polynomial determines the number of possible complex solutions.
* the Remainder Theorem can be used to determine if a factor is a solution.

**Knowledge:**

* **Definitions:** domain, range, solution, vertical line test, horizontal line test, one-to-one functions, composite functions (and notation), inverse, function, relation, independent variables, dependent variables, interval notation, linear, quadratic, cubic, square root, cubed root, reciprocal, absolute value, piece-wise functions (linear & non-linear), parent graphs, vertical and horizontal shifts, reflections (x- and y-axis), vertical and horizontal stretch/shrink, x-intercepts, quadratic functions, polynomial functions, zeros, x-intercepts, roots, complex numbers, complex conjugates, rational zeros, Remainder Theorem, Factor Theorem, Descartes Rule of Signs, synthetic division, degree of a polynomial, Rational Zeros Test, upper and lower bounds
* **Formulas:**
	+ Parent Graphs: $y=x, y=x^{2};y=x^{3};y=x^{4};y=\sqrt{x};y=\sqrt[3]{x};y=\left|x\right|;$

$y=\frac{1}{x};y=A(Bx+C)^{n}+D$y and knowing the variable changes

* + Remainder Theorem: If a polynomial f(x) is divided by (x – k), the remainder is r = f(k).
	+ Factor Theorem: A polynomial f(x) has a factor (x – k) if and only if f(k) = 0.
	+ Rational Zero Test: rational zero =, where p = factor of the constant term, and q = factor of the leading coefficient.
	+ Synthetic Division and Long Division (including expressions with missing terms)
	+ Descartes’ Rule of Signs
	+ Upper and Lower Bounds
* **Graphing Calculator Knowledge:**
	+ Graph a linear or nonlinear equation (y =)
	+ Know how to set appropriate window and use a table
	+ adjust the size of the window (Xmin, Xmax, Ymin, Ymax)
	+ zero and value function under the calc options and/or tracing
	+ Finding minimums and maximums

**Skills:**

*Students will be able to...* **\*Note: Students should know how to graph all of these functions without a graphing utility.**

* find the domain and range of a function in interval notation.
* evaluate a function at a given value.
* use the vertical and horizontal line test to determine if a relation is a function and one-to-one.
* find the inverse of a function algebraically and graphically.
* find the composition of two functions as well as perform the four operations with two functions.
* verify that two functions are inverses using composite functions.
* identify and transform parent graphs.
* describe and apply transformations to parent graphs on a coordinate plane including: vertical and horizontal shifts, vertical stretching and shrinking, horizontal stretching and shrinking and reflections across the x and y axis.
* graph transformations with horizontal reflections/dilations as well as horizontal shift by isolating the variables through factoring.
* graph nonlinear piece-wise functions.
* connect piece-wise functions to domain.
* analyze graph behavior, such as: increasing/decreasing/constant intervals; relative min/max values; intervals where f(x) > 0, f(x) < 0 and f(x) = 0; find output values given input values, and vice versa.
* graph polynomial functions using zeros, degree, end behavior, and leading coefficients.
* prove if a value is a root using any appropriate method (Remainder Theorem, graphing calculator, synthetic division).
* write the complete linear factorization of a rational function.
* use synthetic division as a shortcut to determine roots.
* use the Intermediate Value Theorem to approximate the location of non-rational zeros.
* use Descartes’ Rule of Signs to determine the number of positive, negative and imaginary roots.
* use Upper and Lower Bounds to reduce the number of choices for possible roots.

**Standards:**

**2.1 Numbers and Operations:**

* CC.2.1.HS.F.1: Apply and extend the properties of exponents to solve problems with rational exponents.
* CC.2.1.HS.F.2: Apply properties of rational and irrational numbers to solve real world or mathematical problems.
* CC.2.1.HS.F.3: Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
* CC.2.1.HS.F.4: Use units as a way to understand problems and to guide the solution of multi-step problems.
* CC.2.1.HS.F.5: Choose a level of accuracy appropriate to limitations on measurements when reporting quantities.
* CC.2.1.HS.F.6: Extend the knowledge of arithmetic operations and apply to complex numbers.
* CC.2.1.HS.F.7: Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.

**2.2 Algebraic Concepts**

* CC.2.2.HS.C.1: Use the concept and notation of functions to interpret and apply them in terms of their context.
* CC.2.2.HS.C.2: Graph and analyze functions and use their properties to make connections between the different representations.
* CC.2.2.HS.C.3: Write functions or sequences that model relationships between two quantities.
* CC.2.2.HS.C.4: Interpret the effects transformations have on functions and find the inverses of functions.
* CC.2.2.HS.C.5: Construct and compare linear, quadratic and exponential models to solve problems.
* CC.2.2.HS.C.6: Interpret functions in terms of the situation they model.
* CC.2.2HS.D.1: Interpret the structure of expressions to represent a quantity in terms of its context.
* CC.2.2.HS.D.2: Write expressions in equivalent forms to solve problems.
* CC.2.2.HS.D.3: Extend the knowledge of arithmetic operations and apply to polynomials.
* CC.2.2HS.D.4: Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
* CC.2.2.HS.D.6: Extend the knowledge of rational functions to rewrite in equivalent forms.
* CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
* CC.2.2.HS.D.8: Apply inverse operations to solve equations or formulas for a given variable.
* CC.2.2.HS.D.9: Use reasoning to solve equations and justify the solution method.
* CC.2.2.HS.D.10: Represent, solve and interpret equations/ inequalities and systems of equations/inequalities algebraically and graphically.

**Acceptable evidence**

**Problem Solving Tasks:**

* Piecewise Function Applications
* Linear/Slope Applications
* Football Trajectory

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quia quizzes
* Microsoft OneNote and Forms
* ActivExpressions

Summative Assessment (recommended but up to teacher discretion):

* Quiz 1: Algebra Review, and Functional Algebra (composites and inverses)
* Quest 1
* Quiz 2: End behavior of higher degree polynomials, Factor/Remainder Theorems
* Quest 2

**Suggested Learning Experiences and Instruction**

**Learning Experiences:**

* ActivExpressions for multiple choice review.
* Jeopardy for review.
* Quia or Forms quizzes to review.
* Piecewise functions: practicing graphing linear equations and piecewise functions using whiteboards and have students write their name as a linear and non-linear piece-wise function OR students can use Geometer’s Sketchpad to create their name and specify domains of lines.
* Use Gizmos or Desmos to see how the graph of a rational function is translated (VA, HA, and SA) when different variables are used for the center and leading coefficient.
* In pairs, students create a graph on a graphing calculator.  The students will then exchange their graphs and determine the equation that coincides with the graph.
* Whiteboard games – have each student get a whiteboard.  Using PowerPoint or a flipchart, have one problem on each slide with the solution already prepared.  The students will show the answer and the work on the whiteboard.  For a more cooperative approach have the students work in groups of 3 or 4 and have them do the problems together.  Then use a spinner or dice to select one student per group to show their answer.
* Pass the card – Have enough problems for one per kid or one per pair.  Put the problems on an index card and have the students pass the cards around.  The cards should be numbered and the students should do their work on a blank worksheet in the space that matches the numbers on the cards.
* ActivExpressions – Have the students match the equation to the graph in a multiple choice manner.
* “What equation am I?” Students have 10 questions to guess an equation with transformations (either covered on the board or on a flipchart). Example questions: Are you shifted to the right? Do you have a vertical shift? Are you reflected?
* Precalc Page– Geometer’s Sketchpad Sliders for notes and discovery learning and then the Parent Graphs Power Point for practice.
* Simon Says: kinesthetic game to match your body to the equation.
* Concept Webbing – Review the key concepts of Unit 1 in a concept web.
* 5 minute Writing Prompts: given a function have students write everything they know about a graph/equation.
* Precalc Page – Geometer’s Sketchpad, given the graph write the equation.

**Instructional Strategies:**

* Composition/operations of functions: pass the card activity practicing using all the operations with a few equations.     Challenge: Use critical thinking and work backwards with composition operations. (For example: if f(g(x)= 5 and g(f(x))= 3x, find g(x) and f(x).)
* Interval notation: ( ) more open whereas [ ] are rigid and inclusive.
* Domain: use a memory game to practice finding domain of graphs and domain of equations (each card has either an equation or a graph. the students need to find the domain for each and find the correct pairs for each.)
* Domain: link domain to properties of mathematics that students know so they have a starting place (example: you cannot divide by 0, the square root of a negative number is imaginary).
* For piecewise functions, students should draw the entire graph and then erase using the specified domain if they are having trouble.
* Have the students do activities that require them to know the shape and characteristics of parent graphs.
	+ Matching games or anything that will make them memorize.
	+ Make flash cards of the parent graphs.
* Have the students explore on their own what each of the parameters does to the graph.
	+ Make sure they test positives and negatives, fractions and whole numbers.
* Practice writing the equations of parent graphs given the parent graph and the transformations.
* Use kinesthetic learning and have the students use their arms to show end behavior of even and odd degree polynomials with different leading coefficients.
* Make sure to use all of the terms (zeros, solutions, x-intercepts, and roots) and make sure that the students know they are the same thing.
* Geometer’s Sketchpad with the sliders showing the impact of values on a function.
* Make sure to use all of the terms (zeros, solutions, x-intercepts, and roots) and make sure that the students know they are the same thing!
* Stress that the goal of the rational root theorem and synthetic division is to factor the polynomial into a product of binomials.  This will help the students find the zeros more easily.
* Show the connections between long division, synthetic division, remainder theorem and factor theorem.
* Show students how all the theorems and tests help graph a higher ordered polynomial from scratch, but when practicing, allow students to start with one root from their calculator and then determine the rest through synthetic division and factoring. The focus is understanding roots and factors of the polynomial and how they (along with the degree) determine the graph.

Central Bucks School District – Course of Study

Unit 2: Exponential & Logarithmic Functions

**Desired Results**

**Essential Questions:**

* How do exponential and logarithmic functions relate to real-world situations?
* What is the behavior of an exponential function?
* Why are logarithms used? What problem were they originally intended to solve?
* What is the relationship between exponential and logarithmic functions?
* What is the number e? What does it equal, and why is it important?
* What are the similarities and differences between algebraic rules and logarithmic properties?
* What are the relationships between parent graphs and transformations and the shifts of exponential and logarithmic equations?

**Enduring Understandings:**

*Students will understand that...*

* exponential functions model real-life applications.
* interest is compounded in multiple ways.
* the exponential parent graph shifts in the same manner as the parent graphs and transformations.
* the purpose of logarithms is to solve for a variable in the exponent.
* exponential functions and logarithmic functions are inverses.
* there are restrictions on the domain for logarithmic functions.
* logarithms can be simplified using properties.

**Knowledge:**

*Students will know…*

* **Definitions:** exponential, the number e, logarithmic, change of base, natural logs, common logs, inverses, base 10, compounded, continuously compounded, periodically compounded, principal, interest, annuities, exponential growth, exponential decay
* **Formulas/Properties:**
	+ Change of Base Formula: $log\_{a}b=\frac{logb}{loga}$
	+ Periodic Compounded Interest: $A=P\left(1+\frac{r}{n}\right)^{nt}$
	+ Continuously Compounded Interest: $A=Pe^{rt}$
	+ Population Growth and Decay: $P(t)=Ce^{kt}$
	+ Product Property: $log\_{a}b+log\_{a}c=log\_{a}bc$
	+ Quotient Property: $log\_{a}b-log\_{a}c=log\_{a}\frac{b}{c}$
	+ Power Property: $c log\_{a}b=log\_{a}b^{c}$
	+ Present Value Annuity: $P\_{n}=p\left[\frac{1-(1+i)^{-n}}{i}\right]$
	+ Future Value Annuity: $F\_{n}=p\left[\frac{(1+i)^{n}-1}{i}\right]$
* **Graphing Calculator Knowledge:**
	+ ln/log
	+ e
	+ y= for graphing
	+ exponents and remember ( )

**Skills:**

*Students will be able to…*

* decipher between and use each interest and exponential application formula.
* determine the difference between a present value and future value annuity.
* recognize and transform the parent graph of exponential & log functions.
* use Logarithmic Properties for condensing and expanding logarithmic expressions.
* determine characteristics of the exponential graphs.
* use inverse symmetry to connect exponential and logarithmic graphs.
* evaluate logs and natural logs.
* use the change of base formula to evaluate logs with bases other than 10.
* use log properties to solve exponential equations.
* solve equations with variables in the exponents.

**Standards:**

* CC.2.1. HS.F.2: Apply properties of rational and irrational numbers to solve real world or mathematical problems.
* CC.2.1. HS.F.3: Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays.
* CC.2.1. HS.F.4: Use units as a way to understand problems and to guide the solution of multi-step problems.
* CC.2.1. HS.F.5: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
* CC.2.2. HS.D.9: Use reasoning to solve equations and justify the solution method.
* CC.2.2. HS.D.10: Represent, solve and interpret equations/inequalities and systems of equations and inequalities algebraically and graphically.

**Acceptable evidence**

**Performance Tasks:**

* Population Growth/Decay

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quia quizzes
* ActivExpressions
* Microsoft Forms

Summative Assessment (recommended but up to teacher discretion):

* Quiz 1: Exponential parent graph & transformations
* Quiz 2: Exponential applications:
* Project: Annuities Present and Future Value Mortgages
* Quiz 3: Evaluating and using properties and solving logarithmic equations
* Unit 4 Test

**Suggested Learning Experiences and Instruction**

**Learning Experiences:**

* AIDS simulation activity (on the Precalc page) to introduce exponential functions
* Simulate not paying off a credit card statement to understand compound interest.
* Use Properties of Logs flipchart (located on Intranet PreCalc page).
* Have students research mortgage loans, as well as investments, and the interest rates and years that are available. Using that information, have students calculate information such as what the investment will be worth after a number of years, how long it will take for the investment to double, what the rate must be for the investment to double in x years, etc.
* Have students choose a city, research the population of that city in two given years, then calculate what the population growth rate is, what the population will be in x years, how long it will take for the population to double, etc.
* Have students read <http://home.att.net/~tleary/disme.htm> to learn about how Shakespeare used logarithms when writing some of his sonnets.
* Allow students to create real life scenario projects (ex: buying a car, investing in an IRA, etc) and produce PowerPoints, Photostories, etc.

**Instructional Strategies:**

* Review of exponent and radical rules before beginning.
* Use investment, mortgage, credit cards and real life examples to help students understand the value in investing early and in paying off statements.
* Stress the inverse relationship between exponential and logarithmic functions.
* Practice concepts from Unit 1 such as shifts, transformations, and reflections for graphing.
* Show how expanding and condensing logarithms are opposite operations.
* Stress parenthesis when using a calculator and solving.

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Unit 3: Triangle Trigonometry & The Unit Circle

**Desired Results.**

**Essential Questions:**

* What method is used to solve for missing sides and angles of a triangle?
* When do you use the Law of Cosines vs. Law of Sines and how does that relate to geometric postulates?
* How does the unit circle relate to right triangle trigonometry?
* Why can some triangles be solved and others cannot? And why are there multiple solutions to others?
* What is a radian, and how is it related to degrees?
* In which situations would radians be more effective than degrees?

**Enduring Understandings:**

*Students will understand…*

* that angles can be measured in radians and degrees.
* the unit circle is derived from special right triangles.
* the unit circle is used to determine the six trigonometric functions.
* the Law of Sines and Law of Cosines are used to solve oblique triangles.
* how to use the unit circle to find exact values.

**Knowledge**:

*Students will know…*

* **Definitions**: triangle; right triangle; oblique triangle; angle; unit circle; sine, cosine, tangent; secant, cosecant, cotangent; inverse trigonometric functions; Law of Sines; Law of Cosines; 30-60-90 triangle; 45-45-90 triangle; quadrant; ordered pair; co-terminal angle; reference angle; radian versus degree
* **Formulas:**
	+ Pythagorean Theorem (review): $a^{2}+b^{2}=c^{2}$
	+ Conversion: 1 radian = 1 degree $∙\frac{π}{180°}$ and 1 degree = 1 radian $∙\frac{180°}{π}$
	+ Law of Sines (review): $\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}$$\frac{a}{SinA}=\frac{b}{SinB}=\frac{c}{SinC}$
	+ Law of Cosines (review): $a= \sqrt{a\^2 b^{2}+c^{2}- 2bc(Cos A)}a$$a^{2}=b^{2}+c^{2}-2bc(cosA)$
	+ $45°-45°-90°$ (review): hypotenuse = leg times $\sqrt{2}$
	+ $30°-60°-90°$ (review): hypotenuse = short leg times 2; long leg = short leg times $\sqrt{3}$
	+ $sinθ=\frac{opp}{hyp}$; $cosθ=\frac{opp}{hyp}$; $tanθ=\frac{opp}{adj}$

$cscθ=\frac{hyp}{opp}$; $secθ=\frac{hyp}{adj}$; $cotθ=\frac{adj}{opp}$

* **Graphing Calculator Knowledge:**
	+ Solving for missing sides: Use sin, cos, tan
	+ Solving for missing angles: Use sin-1, cos-1, tan-1
	+ Evaluating cot, sec, and csc using sin, cos, and tan.
	+ Selecting the correct mode and understanding the window of a graph/the axes

**Skills**:

*Students will be able to…*

* solve triangles using Right Triangle rules (Soh-Cah-Toa) (REVIEW).
* solve oblique triangles using the Law of Sines (including the ambiguous case) and the Law of Cosines (REVIEW).
* solve application problems for all triangles using trigonometry.
* generate the unit circle (FROM SCRATCH) values and ordered pairs using special right triangles ($30°-60°-90°$and $45°-45°-90°$).
* convert from degrees to radians and vice versa.
* find the exact value of a trigonometric function using the unit circle.
* find and use reference angles to generate the unit circle.
* relate trig ratios to the coordinates on the unit circle.
* define trig ratios in terms of x, y and r when r is not equal to 1 (non-unit circle).
* determine the sign of the trig ratios in each quadrant using the mnemonic device “All Students Take Calculus”.

**Standards:**

**2.2. Algebraic Concepts**

* CC.2.2. HS.D.8: Apply inverse operations to solve equations or formulas for a given variable.
* CC.2.2. HS.C.7: Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
* CC.2.2. HS.C.9: Prove the Pythagorean identity and use it to calculate trigonometric ratios.

**2.3. Geometry**

* CC.2.3. HS.A.7: Apply trigonometric ratios to solve problems involving right triangles.
* CC.2.3. HS.A.11: Apply coordinate geometry to prove simple geometric theorems algebraically.

**Acceptable evidence**

**Problem Solving Tasks:**

* Baseball & Trees

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quia quizzes
* ActivExpressions

Summative Assessment:

* Quiz 1: Right Triangle Trig and the Law of Sines and Cosines
* Quiz 2: Radians vs. Degrees, Angles on the Coordinate Plane, reference angles
* Quiz 3: Unit Circle Quiz (fill in a blank circle including radians, degrees and coordinates)
* Test on Unit 3

**Suggested learning experiences and instruction**

**Learning Experiences:**

* Make clinometers so students can estimate and use trigonometry to find out the height of a large object outside using the angle and the shadow.
* Have students create a PowerPoint presentation/Photostory about how trigonometry is used in a real life example.
* Have students measure their classmates’ shadow outside and calculate the angle of elevation.
* Use real life oblique triangles for the Law of Sines/Cosines.
* Use sticks/protractors or the tools in ActivInspire to show how two triangles can be created given A-S-S elements.
* Use Inspiration or paper/pencil to develop concept maps for solving different types of triangles.
* Flipchart races for filling in the unit circle (blank unit circle flipchart is on the CFF Precalc page)
* Have students color the unit circle based on reference angles and allow them to determine patterns based on these relationships.

**Instructional Strategies:**

* Use problems that involve rounding as well as exact answers and discuss the value of exact answers.
* To help remember: SohCahToa or $S\frac{0}{h}C\frac{a}{h} T\frac{o}{a}$
* Explain Angle of Elevation and Angle of Depression using Alternate Interior Angles of parallel lines to reinforce that they are equal measure and connect it back to geometry.
* Stress that all trigonometric functions need an angle measurement.
* Stress the importance of drawing diagrams and labeling them.
* Right triangle trig is a review. Applications and oblique triangles should be more of an emphasis as well as applying right triangle trig to the unit circle.
* Make a connection between the trigonometric functions and their inverse functions with the basic inverse functions of addition and subtraction as well as multiplication and division.
* Stress when to use the different methods: right triangle trig, Law of Sines, Law of Cosines and connect everything back to geometry (using concept maps and Inspiration diagrams can help).
* For the Law of Sines (specifically the ambiguous case) and Law of Cosines connect back to the postulates.
* Use the protractor and ruler in ActivInspire to demonstrate the ambiguous case.
* Law of Cosines stress PEMDAS and use a simple algebra equation for comparison.
* Kinesthetic activities for graphing angles on a coordinate plane.
* Make a comparison with degrees/radians to English system/metric conversions.
* Use special right triangles to produce the unit circle.
* Give students several blank unit circles for practice and stress the patterns in the unit circle, specifically using reference angles to help.
* Make connections to geometry throughout the entire unit and provide visuals to enhance learning.

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Unit 4: Graphing Trigonometric Functions

**Desired Results**

**Essential Questions:**

* To what extent are the Unit Circle and trigonometric functions connected graphically?
* Why are some trigonometric functions continuous?
* What is meant by amplitude, period, phase shift, and vertical shift?
* What are the relationships between parent graphs and transformations and the shifts of trigonometric equations?
* Do all trigonometric functions have amplitude and why/why not?
* Do all trigonometric functions have the same period and why/why not?
* What is the relationship between trig functions and their reciprocals?
* What is the relationship between trig functions and their inverses?

**Enduring Understandings:**

*Students will understand that...*

* trigonometric functions can be used to model and solve real-life problems.
* all trigonometric functions can be graphed in degrees or radians.
* amplitude is always positive, although not all trigonometric functions have amplitude.
* period is how long it takes a trigonometric function to complete one cycle.
* tangent and Cotangent have periods that are half the length of the other trigonometric functions.
* each point on a trigonometric graph matches with a point on the Unit Circle.
* there is a phase shift relationship between the sine and cosine curves.

**Knowledge:**

*Students will know...*

* **Definitions**: Period, Amplitude, Phase shift, Vertical shift, Sine, Cosine, Tangent, Cosecant, Secant, Tangent, Cotangent, Arcsin, Arccos, Arctan
* **Formulas:**
	+ y=Asin(Bx-C)+D for various trig functions
		- Amplitude = |A|; if A is negative, the graph is a reflection over the x-axis
		- Period = $\frac{2π}{B}$ or $\frac{360°}{B}$ for sine, cosine, cosecant, and secant
		- Period =$\frac{π}{B}$ or $\frac{180°}{B}$ for tangent and cotangent
		- Phase Shift = $-\frac{C}{B}$
		- Vertical Shift = D
* **Graphing Calculator Knowledge:**
	+ Choosing the appropriate window parameters
	+ Zoom Trig
	+ Choosing the correct mode
	+ Using reciprocal functions to graph

**Skills:**

*Students will be able to…*

* use the unit circle to generate the parent graphs of sine, cosine, and tangent.
* recognize all six trigonometric parent graphs by their shape.
* graph trigonometric functions using transformations including phase shift, vertical shift, reflections, and amplitude.
* determine all characteristics given the equation (amplitude, phase shift, vertical shift)
* determine the starting point and ending point (period + phase shift) of a graph; divide into four labeled, equal intervals (5 points labeled on the axis for at least one cycle).
* write the equation given the characteristics of the trig function or the graph.

**Standards:**

**2.2. Algebraic Concepts**

* CC.2.2.HS.C.2: Graph and analyze functions and use their properties to make connections between the different representations
* CC.2.2.HS.C.4: Interpret the effects transformations have on functions and find the inverses of functions.
* CC.2.2.HS.C.7: Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
* CC.2.2.HS.D.7: Create and graph equations or inequalities to describe numbers or relationships.

**Acceptable Evidence**

**Performance Tasks:**

* Ferris Wheel

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quizzes using Quia and Microsoft Forms
* ActivExpressions

Summative Assessment:

* Quiz 1: Sine and Cosine graphs and transformations
* Unit 6: Test

**Suggested Learning Experiences and Instruction**

**Learning Experiences:**

* <http://illuminations.nctm.org/ActivityDetail.aspx?ID=174> – applet that allows the students to explore the effects of amplitude, period, phase shifts and vertical shifts.
* Whiteboard games – have each student get a whiteboard.  Using PowerPoint or a flipchart, have one problem on each slide with the solution already prepared.  The students will show the answer and the work on the whiteboard.  For a more cooperative approach have the students work in groups of 3 or 4 and have them do the problems together.  Then use a spinner or dice to select one student per group to show their answer.
* Pass the card – Have enough problems for one per kid or one per pair.  Put the problems on an index card and have the students pass the cards around.  The cards should be numbered and the students should do their work on a blank worksheet in the space that matches the numbers on the cards.
* ActivExpressions – Have the students match the equation to the graph in a multiple choice manner.
* <http://www.facade.com/biorhythm/> - biorhythms activity on CFF website Precalc Page.

**Instructional Strategies:**

* Point out from the beginning the unit circle connection to the graphs.
* Point out from the beginning that the transformations from unit 2 still apply to all of the trig graphs.
	+ Now we can see a distinction between horizontal and vertical stretches
* Make sure for sine and cosine, students label 5 key points on the x-axis for one cycle of the graph. For tangent students should at least label 3 key points.
* Steps for graphing:
	+ 1. Determine the period first and then divide that by 4 to get the 4 different sections/axes labels.
		2. Then do the phase shift to give you the left end point and add the shift to all the axes labels in #1.
		3. Then do the vertical shift and draw the new midline (dashed).
		4. Then have the students draw the upper and lower bounds using the amplitude.
		5. Then use the parent graph to plot your 5 points and connect.
* Drawing the parent graph units (and sometimes the parent graph) first is helpful so students can see the changes/transformations
* For csc and sec have the students sketch the sin and cosine graph and then draw the vertical asymptotes.
	+ Cosecant and secant are just the flipped sine and cosine graphs
* Graph tangent using the fact that tangent=sine/cosine
* Have students make flash cards of the parent graphs.
* Make sure to have students match graphs with equations and equations with graphs.
* Stress the difference between the period of a cosine and sine vs. the period of the tangent graph.
* Students should be able to use the following forms and understand the difference: $y=sin⁡(2x-90°)$ and $=sin⁡2(x-45°)$. The latter is easier to determine the characteristics when the GCF exists, but knowing $-\frac{c}{b} $is also useful.

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Unit 5: Trigonometric Identities & Solving Trigonometric Equations

**Desired Results**

**Essential Questions:**

* How can working with trig identities improve your critical thinking skills?
* How can solving trigonometric equations relate to solving polynomial equations?
* Why, when solving trig equations, may your calculator only give you a partially correct answer?
* Why can you have more than one answer when simplifying a trigonometric expression?
* Why can you have more than one answer or no solution when solving a trigonometric equation?

**Enduring Understandings:**

*Students will understand…*

* where Pythagorean Identities come from and how to derive all of them.
* how to complete a trigonometric verification using the identities.
* how to use the unit circle to solve trigonometric equations in specific domains.
* that there is an infinite number of solutions because these graphs are cyclical, therefore answers can be written to include all solutions (ex: ).
* that the algebraic rules/strategies still apply when solving trigonometric equations.

**Knowledge:**

* **Formulas/Identities:**
	+ Reciprocal Identities

$sinθ=\frac{1}{cscθ}$ $cosθ=\frac{1}{secθ}$ $tanθ=\frac{1}{cotθ}$

$cscθ=\frac{1}{sinθ}$ $secθ=\frac{1}{cosθ}$ $cotθ=\frac{1}{tanθ}$

* + Quotient Identities

$tanθ=\frac{sinθ}{cosθ}$ $cotθ=\frac{cosθ}{sinθ}$

* + Pythagorean Identities

$sin^{2}θ+cos^{2}θ=1$ $tan^{2}θ+1=sec^{2}θ$ $1+cot^{2}θ+1=csc^{2}θ$

* + Sum and Difference Formulas:

$$\sin(\left(u+v\right))=\sin(\left(u\right))\cos(\left(v\right))+\cos(\left(u\right))sin⁡(v)$$

$$\sin(\left(u-v\right))=\sin(\left(u\right))\cos(\left(v\right))-\cos(\left(u\right))sin⁡(v)$$

$$\cos(\left(u+v\right))=\cos(\left(u\right))\cos(\left(v\right))-\sin(\left(u\right))sin⁡(v)$$

$$\cos(\left(u-v\right))=\cos(\left(u\right))\cos(\left(v\right))+\sin(\left(u\right))sin⁡(v)$$

$$\tan(\left(u+v\right))=\frac{tanu+tanv}{1-tanutanv}$$

$$\tan(\left(u-v\right))=\frac{tanu-tanv}{1+tanutanv}$$

* + Multiple Angle Formulas:

s$in2u=2sinucosu$ $cos2u=cos^{2}u-sin^{2}u$ $tan2u=\frac{2tanu}{1-tan^{2}u}$

* + Half-Angle Formulas:

$sin\frac{u}{2}=\pm \sqrt{\frac{1-cosu}{2}}$ $cos\frac{u}{2}=\pm \sqrt{\frac{1+cosu}{2}}$

$$tan\frac{u}{2}=\frac{1-cosu}{sinu}=\frac{sinu}{1+cosu}=\pm \sqrt{\frac{1-cosx}{1+cosx}}$$

* **Graphing Calculator Knowledge:**
	+ Choosing the correct mode

**Skills:**

*Students will be able to...*

* substitute the correct trigonometric identities into an expression.
* derive the Pythagorean Theorem identities by dividing by sin2x or cos2x.
* simplify trigonometric expressions using trigonometric identities.
* verify trigonometric identities.
* use standard algebraic techniques to solve trigonometric equations.
* solve trigonometric equations of quadratic type.
* use inverse trigonometric functions to solve trigonometric equations.
* understand when a solution does not exist to a trig equation.
* use sum and difference formulas to evaluate trigonometric functions, verify identities, and solve trigonometric equations.
* use multiple-angle formulas to rewrite and evaluate trigonometric functions.
* use half-angle formulas to rewrite and evaluate trigonometric functions.

**Standards:**

**2.2 Algebraic Concepts**

* CC.2.2.HS.D.1: Interpret the structure of expressions to represent a quantity in terms of its context.
* CC.2.2.HS.D.9: Use reasoning to solve equations and justify the solution method.
* CC.2.2.HS.C.7: Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
* CC.2.2.HS.C.9: Prove the Pythagorean identity and use it to calculate trigonometric ratios.

**Acceptable evidence**

**Problem Solving Tasks:**

* Verifying & Solving

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quia quizzes
* ActivExpressions
* Microsoft Forms and OneNote

Summative Assessment:

* Mini-quiz on fundamental identities and deriving identities
* Quiz 1: Verifying
* Quiz 2: Solving
* Unit 6 Test

**Suggested Learning Experiences and Instruction**

**Learning Experiences:**

* [www.explorelearning.com](http://www.explorelearning.com) - Simplifying Trigonometric Expressions
* Flash Cards for identities
* Power Point Jeopardy game
* Randomize the steps used in verifying a trig identity.  Give groups of students all necessary steps and have them put them in the correct order, like a puzzle.
* ActivExpressions are useful for quick memorization but also to assess confidence/studying
* Quia flash cards or memory games
* Podcasts/Photostories with rhyming, poems or pictures to help remember the identities or ways to solve

**Instructional Strategies:**

* Provide several opportunities for students to practice and make sure students memorize the identities.
* Derive Pythagorean identities from Pythagorean Theorem
* Show students the main Pythagorean Identify sin2x+cos2x=1 and how to find the other two by dividing by sin2x or cos2x.
* Derive reciprocal and quotient identities from Unit Circle
* Give strategies for verifying identities:
	+ Knowledge of fundamental trigonometric identities is essential.
	+ Work on more complicated side first.
	+ Look to change everything into sine and cosine as a first step.
	+ Review factoring with trigonometric functions.
	+ Substitution using identities.
* Give strategies/tips for solving trigonometric equations:
	+ Make sure students solve for all quadrants.
	+ Utilize “All Students Take Calculus” to determine where functions are positive on the unit circle.
	+ Have students answer in both degrees and radians.
	+ Have students determine whether solutions are extraneous by connecting solutions to the graph/unit circle.
	+ Utilize proper notation, for answers including ALL solutions, such as (ex: $\frac{π}{6}\pm 2πk$).

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Unit 6: Conic Sections

**Desired Results**

**Essential Questions:**

* What is the difference between the four conic sections?
* Where are conic sections found in the real world?
* Why is it important to understand conic sections?
* How do the graphs of the four conic sections differ?

**Enduring Understandings:**

*Students will understand that...*

* conic sections are used regularly in real life.
* the graphs and equations of the four conic sections are different.
* you must find all the necessary components to graph the equation.

**Knowledge:**

*Students Will Know…*

* **Definitions**: circle, ellipse, parabola, hyperbola, foci, asymptotes, minor axis, major axis, semi-minor axis, semi-major axis, conjugate axis, transverse axis, general form, standard form, locus, radius, concentric circles, vertices, directix
* **Formulas:**
	+ Circle standard form: $\left(x-h\right)^{2}+\left(y-k\right)^{2}=r^{2}$
	+ Circle general form: $x^{2}+y^{2}+Dx+Ey+F=0$
	+ Parabola standard form: $\left(y-k\right)^{2}=4p\left(x-h\right)$ or $\left(x-h\right)^{2}=4p\left(y-k\right)$
	+ Parabola general form: $y^{2}+Dx+Ey+F=0$ or x2 + Dx + Ey + F = 0
	+ Ellipse standard form: $\frac{\left(x-h\right)^{2}}{a^{2}}+\frac{\left(y-k\right)^{2}}{b^{2}}=1$ $\frac{\left(y-k\right)^{2}}{a^{2}}+\frac{\left(x-h\right)^{2}}{b^{2}}=1$

or

* + Ellipse general form: $Ax^{2}+By^{2}+Dx+Ey+F=0;\left(A\ne B\right)$
	+ Hyperbola standard form: $\frac{\left(x-h\right)^{2}}{a^{2}}-\frac{\left(y-k\right)^{2}}{b^{2}}=1$ $\frac{\left(y-k\right)^{2}}{a^{2}}+\frac{\left(x-h\right)^{2}}{b^{2}}=1$

or

* + Hyperbola general form: $Ax^{2}+By^{2}+Dx+Ey+F=0;\left(A<0 or B<0, not both\right)$

**Skills:**

*Students will be able to…*

* determine the difference between each of the conic graphs and their equations.
* determine the orientation of the various conic sections.
* use standard and general forms of a circle, ellipse, parabola and hyperbola.
* find the center of a circle, ellipse, and hyperbola.
* convert between standard and general form for each conic section.
* graph the four conic sections.
* identify the focus, vertex, directrix, and axis of symmetry of a parabola.
* identify the center, foci, vertices, and length of major and minor axes of an ellipse.
* identify the center, foci, vertices and asymptotes (in point slope form) of a hyperbola.
* Write equations of the various conic sections given a graph or a locus of points (vertices, foci, etc).

**Standards:**

**2.2. Algebraic Concepts**

* CC.2.2.HS.C.3: Write functions or sequences that model relationships between two quantities.

**2.3. Geometry**

* CC.2.3.HS.A.10: Translate between the geometric description and the equation for a conic section.

**Acceptable evidence**

**Performance Task: Bridge the Gap**

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quia quizzes
* ActivExpressions
* Micrsoft forms and OneNote

Summative Assessment:

* Daily quizzes
* Quiz on sections 10.1-10.2
* Unit Test on sections 10.1-10.5 (new material: circles, ellipses, parabolas, hyperbolas)

**Suggested Learning Experiences and Instruction**

**Learning Experiences:**

* ActivExpressions and Microsoft Forms for multiple choice review
* Jeopardy for review
* Quia quizzes to review
* Demonstrate how each type is a section of a cone.
* Demonstrate the importance of the foci for an ellipse using string.
* Have students explore conic sections using [www.explorelearning.com](http://www.explorelearning.com).

**Instructional Strategies:**

* Show the connection between the Pythagorean Theorem and the Conic Sections.
* Show the connection between an ellipse and a circle.
* Have students create a graphic organizer for all of the conic sections and their components.

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Unit 7: Parametric and Polar Equations

**Desired Results**

**Essential Questions:**

* What advantages does parametric mode have over rectangular/function mode?
* What calculus concepts can we apply to parametric equations?
* How are equations used to model motion of projectiles?
* How do you graph a point on a polar coordinate system?
* How do you graph an equation on a polar coordinate system?
* How do you convert from polar form to rectangular form and vice versa?

**Enduring Understandings:**

*Students will understand that...*

* Parametric functions are useful for modeling situations involving position, velocity, and acceleration.
* Parametric functions are useful for analyzing curves that are not rectangular functions.
* Polar coordinates are used to graph, two dimensionally, the coordinates and equations that occur on a sphere.
* The relationships between polar and rectangular coordinates originate from trigonometric concepts.

**Knowledge:**

*Students Will Know…*

* **Definitions:** parametric equations, parameter, arc length, position, velocity, acceleration, speed, displacement, total distance traveled, polar, polar coordinate, limaçon, cardioid, rose, Spiral of Archimedes
* **Formulas/Rules:**
	+ Parametric Equations of a Line: $x=x\_{1}+ta\_{1}$

$$y=y+ta\_{2}$$

* + Converting polar coordinates to rectangular: $x=rcosθ$, $y=rsinθ$
	+ Converting rectangular coordinates to polar: $r^{2}=x^{2}+y^{2}$, $θ=tan^{-1}\frac{x}{y}$
	+ Polar graphs:
	+ Lines: horizontal: $rcosθ=a$; vertical: $rsinθ=b$; through origin: $θ=θ\_{0}$
	+ Circles:
		- Centered at origin, radius = *a*: $r=a$
		- Centered on the x or y axis, through the origin, radius = *a*: $r=\pm 2acosθ$ or $r=\pm 2asinθ$
	+ Cardioid:
		- X-axis symmetry: $r=a\pm acosθ$
		- Y-axis symmetry: $r=a\pm asinθ$
		- Limaçon: $r=a\pm bcosθ$ or $r=a\pm bsinθ$ (same symmetry as cardioids)
		- Inner loop: $\left|\frac{a}{b}\right|<1$
		- Dimpled: $1<\left|\frac{a}{b}\right|<2$
		- Convex: $2\leq \left|\frac{a}{b}\right|$
	+ Lemniscates:
		- Along x-axis: $r^{2}=a^{2}cos2θ$
		- Along y-axis: $r^{2}=-a^{2}cos2θ$
		- Along line y = x: $r^{2}=a^{2}sin2θ$
		- Along line y = -x: $r^{2}=-a^{2}sin2θ$
	+ Roses: $r=asin⁡(nθ)$ or $r=acos⁡(nθ)$
		- # petals = n if n is odd
		- # petals = 2n if n is even
* **Graphing Calculator Knowledge:**
	+ Use basic trigonometric functions
	+ Students should be able to graph the classical curves

**Skills:**

*Students will be able to…*

* Graph a parametric function with or without technology.
* Model the motion of a projectile using parametric equations
* Graph points, circle, and lines in polar form.
* Name the coordinates of a point in polar form in multiple ways.
* Identify and graph the classical curves (limaçon, cardioid, rose, and Spiral of Archimedes).
* Convert coordinates and equations from polar form to rectangular form.
* Convert coordinates and equations from rectangular form to polar form.
* Rewrite a point in polar coordinates using alternate values of *r* or*θ*.

**Standards:**

**2.1 Numbers and Operations:**

* CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems.

**2.2. Algebraic Concepts:**

* CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
* CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials.
* CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
* CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method.
* CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
* CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
* CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
* CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
* CC.2.2.HS.C.6 Interpret functions in terms of the situations they model.
* CC.2.2.HS.C.7 Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
* CC.2.2.HS.C.8 Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

**Acceptable evidence**

**Performance Task:**

* Practice multiple choice questions
* Practice free response questions

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessment:

* Warm-ups
* Exit Slips
* Keeping a notebook
* Homework
* Class work
* Quia quizzes
* ActivExpressions

Summative Assessment:

* Daily quizzes
* Quiz section 10.6
* Unit Test on section 10.6-10.8

**Suggested Learning Experiences and Instruction**

**Learning Experiences:**

* Jeopardy for review.
* Quia quizzes to review.
* Use a sphere to show the relationship between graphing on a sphere and graphing on a 2D polar system.
* Explore the special curves using the graphing calculator/or graphing by hand.

**Instructional Strategies:**

* Have the students make flash cards with the different formulas.
* Demonstrate and have students draw pictures for real-work applications.
* Connect the formulas to right triangle trigonometry.
* Superimpose a rectangular plane on top of a polar plane.

Central Bucks School District – Course of Study

Unit 8: Limits

**Desired Results**

Essential Questions:

* How is a rational function different from a polynomial function (including the graphs)?
* What is the difference between asymptotes and holes?
* How can factoring and division be used to help find asymptotes?
* How do rational functions connect to the basic laws of mathematics (for example, not being able to divide by zero)?
* What is the procedure for graphing a rational function without a graphing utility?
* What is the difference between a limit value and the function value?
* What is the connection between the domain of a function and its discontinuities?
* What is the connection between limits at infinity and end behavior?

**Enduring Understandings:**

*Students will understand that…*

* a rational function is a division of two polynomial functions.
* the domain of a rational function (excluding radicals) includes all real numbers except those that make the denominator zero.
* a rational function can have multiple vertical asymptotes.
* a rational function can have either a horizontal asymptote or a slant asymptote, but not both.
* a hole is a removable discontinuity whereas asymptotes are infinite discontinuity
* if a limit and a function value are the same, then the function is continuous at that point.
* a limit can exist at a point of discontinuity.
* a limit is the intended y value of a function as the x value gets infinitely close to the given x value.

**Knowledge:**

*Students will know…*

* Definitions: rational function, discontinuity, removable discontinuity, infinite discontinuity, vertical asymptote, horizontal asymptote, slant asymptote, holes, limit, left hand limit, right hand limit, continuity, discontinuity, point discontinuity, jump discontinuity, infinity, non-removable discontinuity
* Formulas/notations:
	+ Rational Function: $f\left(x\right)=\frac{N\left(x\right)}{D(x)}, D(x)\ne 0$
	+ Limit Notation: $\lim\_{x\to c}f\left(x\right)=L$
	+ Left and Right Hand Limits: $\lim\_{x\to c^{+}}f\left(x\right)=L$; $\lim\_{x\to c^{-}}f\left(x\right)=L$
* **Graphing Calculator Knowledge:**
	+ Students should be aware that calculators do not show holes
	+ Change to dot mode in order to avoid two parts of a graph from being connected at a vertical asymptote (if you have an old operating system)

**Skills:**

*Students will be able to…*

* determine all areas of discontinuity (asymptotes and holes) of a rational function.
* connect discontinuity to domain of rational functions.
* find the x- and y-intercept(s) of rational functions.
* graph any rational function.

**\*Note: Students should know how to graph all of these functions without a graphing utility.**

* evaluate one sided limits.
* evaluate if a limit exists.
* evaluate a limit algebraically (using substitution or simplification).
* evaluate a limit graphically.
* evaluate limits involving infinity.
* evaluate limits of piece-wise functions.
* determine if a function is continuous at a specific point.

**Standards:**

* 1. **Numbers and Operations**
* CC.2.1.HS.F.1: Apply and extend the properties of exponents to solve problems with rational exponents.
* CC.2.1.HS.F.2: Apply properties of rational and irrational numbers to solve real world or mathematical problems.
* CC.2.1.HS.F.3: Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
* CC.2.1.HS.F.4: Use units as a way to understand problems and to guide the solution of multi-step problems.
* CC.2.1.HS.F.5: Choose a level of accuracy appropriate to limitations on measurements when reporting quantities.
* CC.2.1.HS.F.6: Extend the knowledge of arithmetic operations and apply to complex numbers.
* CC.2.1.HS.F.7: Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.

**2.2 Algebraic Concepts**

* CC.2.2.HS.C.1: Use the concept and notation of functions to interpret and apply them in terms of their context.
* CC.2.2.HS.C.2: Graph and analyze functions and use their properties to make connections between the different representations.
* CC.2.2.HS.C.3: Write functions or sequences that model relationships between two quantities.
* CC.2.2.HS.D.3: Extend the knowledge of arithmetic operations and apply to polynomials.
* CC.2.2HS.D.4: Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
* CC.2.2.HS.D.6: Extend the knowledge of rational functions to rewrite in equivalent forms.
* CC.2.2.HS.D.7: Create and graph equations or inequalities to describe numbers or relationships.
* CC.2.2.HS.D.9: Use reasoning to solve equations and justify the solution method.
* CC.2.2.HS.D.10: Represent, solve and interpret equations/ inequalities and systems of equations/inequalities algebraically and graphically.

**Acceptable evidence of learning other than Performance Tasks:**

Formative Assessments:

* Warm-ups
* Checkpoint assignments
* Read and Take Notes (RTN)
* Classwork
* Homework
* Exit slips
* Quia & Quizzes in Microsoft Forms
* ActivInspire Activities
* ActivExpressions
* Possible Projects

Summative Assessments:

* Quiz 1: Graphing Rational Expressions
* Quiz 2: Determining Limits Graphically
* Unit Test

**Instructional Strategies:**

* Review what a rational number is and then compare to a rational function
* Have students graph $\frac{1}{x}$. Discuss what asymptotes are and where they are.
	+ Discuss what the denominator cannot equal
		- What happens here on the graph? (Vertical Asymptote)
* Graph $y=\frac{1}{x-2}$.
	+ Discuss where the asymptotes are here.
* Graph $y=\frac{x-2}{x^{2}-4}$
	+ What can we do? (factor)
	+ What could happen? $(\frac{0}{0})$
		- This causes a hole. Explain this and how to find the coordinates of the point.
		- Focus on the difference of point discontinuity (hole) vs. infinite discontinuity (VA)
* Practice holes and VAs
* Discuss types of Horizontal Asymptotes
	+ Only worry about the term with the highest exponent in both the numerator and denominator.
		- Explain that this is because horizontal asymptotes are as x approaches infinity (connect to end behavior)
	+ Higher exponent in denominator 🡪 asymptote at y = 0
	+ Same exponent - asymptote at ratio of leading coefficients
* Slant asymptotes
	+ Occur when there are no HAs
	+ Denominator exponent is exactly 1 less than numerator exponent
	+ To find: divide numerator by denominator, ignoring any remainder.
		- Use synthetic division when denominator is linear
		- Use long division when denominator is non-linear
* Practice determining when a function has slant asymptotes.
* Practice Graphing
	+ Plot zeros, holes, VAs, SAs, and HAs.
* When does a limit exist? http://www.calculus-help.com/tutorials/
* Create a flowchart of methods for finding a limit both algebraically and graphically.
* Limits involving infinity: <http://archives.math.utk.edu/visual.calculus/>
* Use ActivExpressions and the Promethean board to help with the visualization and assessing.
* Have students sketch a graph given limits and conditions.
* To determine the quadrant where the graph exists, choose a point to the left and right of each VA and plot it. The entire part of the graph will exist in this quadrant.
* Model real-world applications using rational functions.

Learning Experiences:

* Use flow charts/concept mapping to organize finding the different discontinuities in rational functions. Inspiration can be useful for these graphic organizers.
* Given certain parameters have students graph (on sketchpad or their calculator) a rational function that fits all specifications.
* Students can use Inspiration or pencil/paper to generate concept maps for rules and relationships between asymptotes and holes.

Appendix – Table of Contents for Teacher Resources

The Following Documents Are Available On the District Intranet Site Problem Solving Questions:

Unit 1: Piecewise Function Applications & Linear/Slope Applications

Unit 2: Football Trajectory

Unit 3: Rational Functions

Unit 4: Population Growth/Decay

Unit 5: Baseball & Trees

Unit 6: Ferris Wheel

Unit 7: Verifying & Solving

Unit 8: Roulette

The following website has lots of instructional technology examples and activities for your use as well: <http://www1.cbsd.org/curriculum/cff/math/Pages/Pre-Calculus.aspx>

Again, the following site is useful for the statistics supplement in Unit 8: <http://college.hmco.com/mathematics/larson/precalculus_limits/1e/resources/ab.html>

The following documents are available through your building coordinator until they can be password protected on the intranet site.

Core 1: Solving Trigonometric Equations

Core 2: Graphical and Algebraic Analysis of Limits

Core 3: Conic Sections

Final Exam Review

Final Exam